Int. J. Heat Mass Transfer. Vol. 33, No. 5, pp. 869–875, 1990
Printed in Great Britain

0017-9310/90 $3.00+0.00

Confined natural convection due to lateral heating in a stably stratified solution

J. LEE,† M. T. HYUN‡ and Y. S. KANG†

† Department of Mechanical Engineering, Yonsei University, Seoul 120-749, Korea
‡ Department of Engine Technology, Cheju University, Cheju-do 690-121, Korea

(*Received* 28 *February* 1989 *and in final form* 19 *July* 1989)

Abstract—Experimental investigations have been conducted to study steady natural convection of a stably stratified salt–water solution due to lateral heating in a rectangular enclosure of aspect ratio 3.0. Depending on the buoyancy ratio, N, which represents the relative magnitude of solutal stratification to thermal buoyancy, four distinct flow regimes are observed—unicell flow regime for $N < 10$, simultaneously formed layer flow regime for $10 \leq N < 40$, successively formed layer flow regime for $40 \leq N < 55$ and stagnant flow regime for $N \geq 55$. Formation and growth of the layered flow structure is visually observed and described, with the corresponding temperature and concentration distributions in each layer.

1. INTRODUCTION

THERMOSOLUTAL convection has been receiving much attention over the past two decades due to its relation to many transport processes in nature and technology such as atmospheric flows, drying processes, liquid gas storage and materials processing. Recently with the need for ever more perfect crystals from the high technology industries, much attention is being given to the role of natural convection in crystal growth systems because the transport processes in the fluid phase during the growth of a crystal has a profound influence on the structure and quality of the solid phase.

It was pointed out that various modes of convection are possible depending on how both gradients are oriented relative to each other as well as to the body force [1, 2]. Most of the earlier works on thermosolutal convection have been given to the unbounded regions in which both gradients are parallel to the body force direction, in order to explain some unusual oceanographic phenomena such as salt fingers and salt fountains in deep sea or ocean. Turner [3] has summarized much of the previous works. Recent research has been carried out on natural convection in confined stratified fluids either with imposed lateral temperature gradients [4–7] or with heating from below [8, 9], and recently a few works [10–12] have been carried out on the confined natural convection with the combined horizontal temperature and concentration gradients which has an important bearing on the crystal growth system.

In this work, we are interested in examining the various flow regimes appearing in confined stratified fluids due to lateral heating. This problem has been receiving attention since the initiation of research on the mechanism of layered structures both in temperature and salinity in many parts of the world's oceans. Stern and Turner [13] carried out experiments with salt and sugar solutions and found that layers of constant density may be formed successively due to the salt-fingering process. Another mechanism in the establishment of the layering phenomena is the occurrence of layered flow patterns in density stratified fluids due to the lateral temperature gradient. Thorpe *et al.* [4] observed experimentally that cells are formed successively from the top and bottom boundaries in the stratified brine solution due to the heating of vertical boundaries and explained from the linear stability analysis that the horizontal gradients of salinity and temperature may be responsible for that as noted by Stern [14]. Chen *et al.* [5] and Wirtz *et al.* [6] also verified the existence of layered flow structures in their experiment and indicated two mechanisms of layered flow formation in terms of Rayleigh number based on the length scale which is the potential rise of a heated element of fluid in an initial density gradient. At the subcritical Rayleigh number the layers begin to appear from the horizontal boundaries and develop successively inward towards the mid-region. After sufficient time, however, the cells decrease in number and in some cases only two well-established cells remain, each near the top and bottom boundaries. At supercritical Rayleigh number, on the other hand, the cells occur simultaneously along the heated walls, grow laterally to reach the opposite wall and ultimately produce the layered flow system. The fully established layered flow pattern is stable and persists for a long time.

Earlier works mentioned above are mainly concerned with the mechanism of the layered flow structure, and thus, their experiments are restrictive. In other words, the layered flow structures are not the whole picture of this problem, and there may occur different flow patterns not observed in the earlier works. In addition, the above results are not strictly steady-state observations because the initially imposed stratification changes due to the cellular fluid

NOMENCLATURE

Ar	aspect ratio, H/L	Sc	Schmidt number, ν/D
ΔC	concentration difference [wt%]	ΔT	temperature difference [°C]
D	diffusivity of salt [$m^2 s^{-1}$]	x	horizontal coordinate [m]
g	acceleration due to gravity [$m s^{-2}$]	y	vertical coordinate [m].
H	height of the enclosure [m]		
L	width of the enclosure [m]		
Le	Lewis number, α/D	Greek symbols	
N	buoyancy ratio, $\bar{\beta}\Delta C/\beta\Delta T$	α	thermal diffusivity [$m^2 s^{-1}$]
Pr	Prandtl number, ν/α	β	thermal expansion coefficient [$°C^{-1}$]
Ra_S	solutal Rayleigh number, $g\bar{\beta}\Delta C H^3/\nu\alpha$	$\bar{\beta}$	solutal expansion coefficient [$wt\%^{-1}$]
Ra_T	thermal Rayleigh number, $g\beta\Delta T H^3/\nu\alpha$	ν	kinematic viscosity [$m^2 s^{-1}$].

motion. Thus, in the present work experimental investigations are made to study the steady natural convection of a stratified salt–water solution due to lateral heating in a rectangular enclosure of aspect ratio 3.0. The primary objective is to obtain the steady global flow patterns in detail that will possibly appear in the enclosure with the corresponding temperature and concentration gradients under various parametric conditions. It was possible to obtain the steady-state result by replacing the top and bottom boundaries by membranes with salt–water reservoirs which are maintained at constant, but different concentrations corresponding to the imposed initial stratification. This means that the horizontal boundary condition is changed to constant concentration conditions instead of zero mass flux adopted by earlier works. It proves that the parameter, the buoyancy ratio, which is the ratio of solution stratification to thermal buoyancy, is very useful in describing the various flow regimes observed.

2. EXPERIMENTS

A schematic diagram of the experimental set-up is given in Fig. 1. The apparatus consists of a rectangular enclosure and supporting reservoirs. The test section is a horizontally placed rectangular enclosure 120 mm high, 40 mm wide and 100 mm deep. Two opposite vertical walls (120 mm × 100 mm) are made of aluminum plates and kept at different but constant temperatures by circulating water adjusted to the experimental conditions from the two constant temperature baths. Five thermocouples are installed on the surface of each isothermal wall in order to check the temperature of the vertical boundaries. The horizontal walls consist of membranes with supporting reservoirs, which are worked out to impose the constant concentration condition at both top and bottom walls using the osmosis process across the membrane. The membrane adopted in the present test is 250 PM type made by Enka (Germany). The whole set-up except the end walls is made of 10 mm thick acryl plate and is enclosed by styrofoam of 30 mm thickness to minimize the heat loss from the system to the surroundings.

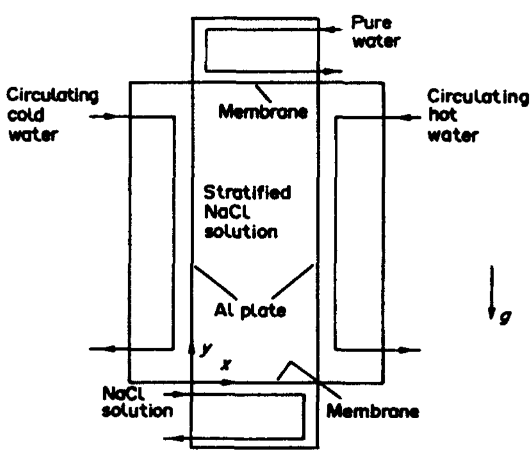


FIG. 1. Schematic diagram of the experimental set-up.

Pure water and a salt solution are used as the working fluids. The salt–water solution is previously stratified in the enclosure by Oster's method [15]. Pure water is supplied into the top supporting reservoir whereas a salt–water solution is poured into the bottom supporting reservoir according to the imposed initial salt stratification in the enclosure. The fluid in the enclosure is kept still for about 3 h so that the concentration profile may smooth out by diffusion and a linear salinity distribution is clearly obtained. After the linear salinity profile is established in the test section, the experiment is started by allowing heated water at a predetermined temperature to circulate through the passages in one of the aluminum plates while the other is maintained at that of the fluid in the tank. In this experiment, the hot wall is on the right-hand side of the enclosure. It takes 4–5 h to reach the steady state, and since then the flow pattern did not change within 24 h. The temperature and concentration profiles are obtained at about this time.

The temperature distributions in the enclosure are measured using a continuously traversing probe made by a copper–constantan thermocouple at the mid-section of the test section. The outputs of all thermocouples are recorded on a data logger and multi-pen recorder. In order to determine the concentration profiles, an electroconductivity probe was constructed using platinum wire, because the point electroconductivity probe is generally chosen as the method of measuring salt concentration at points in a confined enclosure [16]. We found, however, that the calibration changes due to the increasing probe resistance during the measurement. Both gain and reference, thus, had to be adjusted between calibrations in order to make the second calibration meaningful. Therefore, the sample extraction method was used here and for this purpose holes of 1 mm diameter at intervals of 5 mm in the vertical direction are made in the middle of the back wall. Through the hole a minute amount of solution is extracted by fine needles of 0.3 mm i.d. and its refractive indices are read through a refractometer (Atago, Abbe-Refractometer). This procedure is repeated at a point and each refractive index is averaged. The deviation from the average values is within $\pm 5\%$. As the needle is inserted slowly and normal to the flow direction, the disturbance of the flow field due to solution extraction is negligible. Flow patterns in the enclosure are examined by visualizing them with dye injection through the hole in the back wall using camera and video systems.

The experiments cover the range of $Pr = 4.5$–6.4, $Sc = 454$–640, $Ra_T = 9.2\times10^7$–1.0×10^9 ($\Delta T = 2.5$–15.7°C), $Ra_S = 4.2\times10^9$–1.4×10^{10} ($\Delta C = 5$–15 wt%) and $N = 7.1$–98.1. All fluid properties [17] are evaluated at the average value of both the temperature and concentration of each end wall.

3. RESULTS AND DISCUSSION

Flow visualization shows four types of global flow patterns in the enclosure according to the interaction of given salt stratification and lateral heating. In terms of buoyancy ratio, N, which represents the relative magnitude of salt stratification to lateral heating, there appears a unicell flow regime for $N < 10$, a simultaneously formed layer flow regime for $10 \leqslant N < 40$, a successively formed layer flow regime for $40 \leqslant N < 55$ and a stagnant flow regime for $N \geqslant 55$. Figure 2 shows the typical formation of these flow structures with time and the corresponding temperature and concentration profiles are shown in Figs. 3–6.

When the thermal buoyancy due to lateral heating is very weak compared to the given solutal stratification ($N \geqslant 55$), there appears no motion at all from the beginning in the enclosure due to the strong solutal stratification (stagnant flow regime). In this flow regime, as shown in Fig. 3, the initial concentration distribution remains almost unchanged and the temperature variation is strictly linear in the horizontal direction indicating that heat transfer is by conduction only.

When lateral heating becomes effective ($40 \leqslant N < 55$), rolls begin to appear from the top and bottom boundaries. They grow laterally and develop successively inward with time (Fig. 2(a), successively formed layer flow regime). But the final layer development is restricted near the top and bottom regions because layers merge into one another in the mid-region, and in the meantime they disappear. Thus there exists no layered flow structure in the mid-region and the fluid remains almost stagnant there. It is seen from Fig. 4 that in each layer the temperature profile is weakly stratified while in the mid-region, it is unstably stratified. The concentration in each layer is nearly uniform except near the top and bottom walls and in the mid-region, it is stably stratified.

When lateral heating is more effective compared to the salt stratification ($10 \leqslant N < 40$), rolls form simultaneously all along the heated wall, grow laterally with time and ultimately produce the layered flow structure (Fig. 2(b), simultaneously formed layer flow regime) as observed by Chen *et al.* [5]. After the layers are established in the enclosure, some adjustment occurs between layers by merging of the layers and finally the layer size is rather increased while the number is decreased. Since the layer merging occurs more quickly as the buoyancy ratio decreases, the length scale suggested by Chen *et al.* [5] is meaningful at the stage of the initial formation of cells and for a relatively larger buoyancy ratio ($N > 30$). The number of layers appearing in the enclosure depends on the relative magnitude of both gradients. For the same solutal stratification more layers appear in the enclosure for the smaller temperature differences. In each test, the buoyancy flow is counterclockwise since the hot wall is on the right-hand side of the enclosure. The interface of each layer slightly slopes downward as it approaches the cold wall because the density increases due to the cooling effect of the end wall. In this flow regime, the temperature variation in each layer is large in the vertical direction and the concentration is nearly equal as shown in Fig. 5. The temperature varies smoothly at the interface of each layer while the concentration changes rapidly at the interface of fast moving layers as shown in Figs. 4 and 5. This phenomenon is largely due to the difference of the diffusivities of heat and salt ($Le \simeq 100$). Heat with larger diffusivity diffuses at the interface fast enough to make the temperature variation smooth even in the case of fast moving layers, but the salt with much lower diffusivity cannot diffuse so rapidly as to have the smooth concentration variations at the interface when the layered fluid moves fast.

When the thermal buoyancy due to lateral heating is dominant ($N < 10$), the flow is initially formed as a layered structure, but as time goes on the layers merge and finally thermally-driven unicellular fluid motion sustains (Fig. 2(c), unicell flow regime). As shown in Fig. 6, the temperature distribution in this case is very similar to those of pure thermal convection [18, 19] and the initially imposed linear concentration profile becomes uniform due to convective mixing.

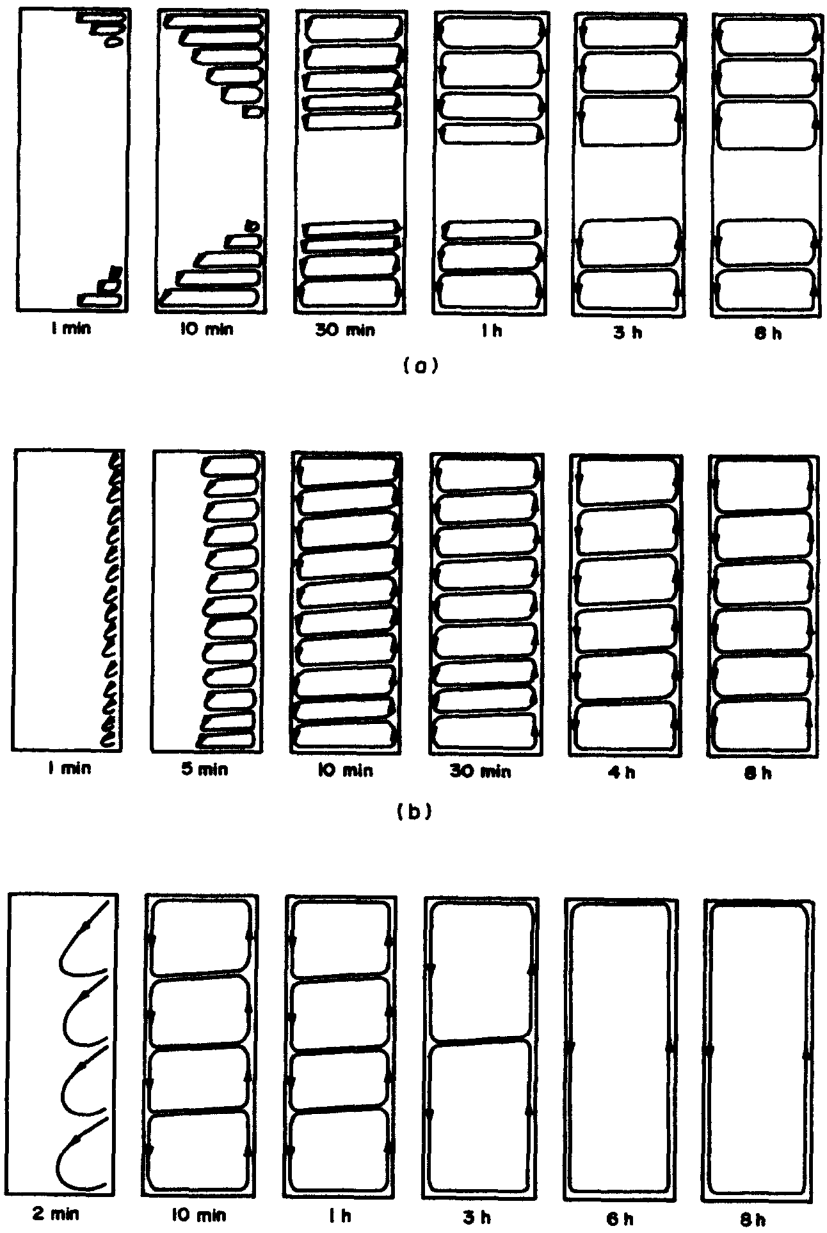


Fig. 2. Formation of various flow structures with time: (a) successively formed layer flow for $Ra_T = 9.20 \times 10^7$ and $N = 50.1$; (b) simultaneously formed layer flow for $Ra_T = 2.49 \times 10^8$ and $N = 18.2$; (c) unicell flow for $Ra_T = 4.61 \times 10^8$ and $N = 9.9$.

Each flow regime is represented in terms of thermal and solutal Rayleigh numbers (Fig. 7(a)) and buoyancy ratio (Fig. 7(b)), though the demarcation in the figure is not strictly applicable with confidence due to lack of sufficient experimental data. From the figure, it can be seen that for a given solutal Rayleigh number the flow pattern changes from region I to region IV with an increasing thermal Rayleigh number, and there is a certain range of the buoyancy ratio over which each flow regime exists. Flow regimes I and IV are not observed in Chen *et al.*'s experiment [5], because they are concerned only on a layered flow structure. But, considering that their experimental ranges ($35 < N < 110$) lie in the right portion of Fig. 7(b), it could be expected that if the experiments are extended to much higher thermal Rayleigh number (i.e. much lower buoyancy ratio), there would appear a unicell flow regime due to the strong thermal buoyancy. We notice that there occurs no motion in the present experiment for $N \geqslant 55$ while layered flow structures are observed up to $N = 110$ in their experiment. The main reason is supposed to be the difference of the top and bottom boundary conditions. In Chen *et al.*'s experiment [5], the initial stratification is apt to change due to the fluid motion and once changed it does not give the same retardation effect

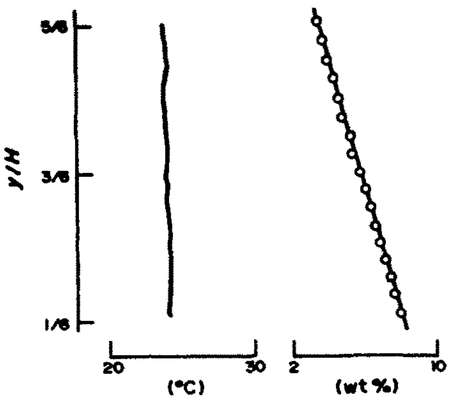


FIG. 3. Vertical temperature and concentration profiles at $x/L = 1/2$ for stagnant flow regime: $Ra_T = 1.43 \times 10^8$, $N = 58.9$.

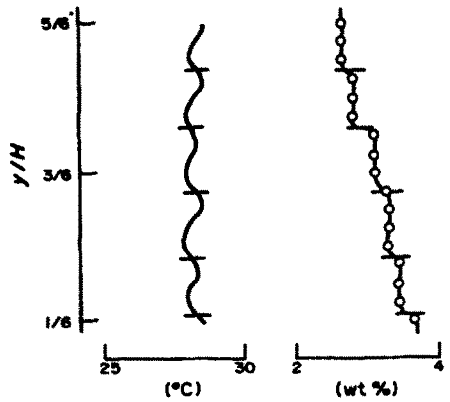


FIG. 5. Vertical temperature and concentration profiles at $x/L = 1/2$ for simultaneously formed layer flow regime: $Ra_T = 1.42 \times 10^8$, $N = 32.9$.

on the fluid motion as it has in the beginning, while in the present experiment, the salt stratification is being kept constant in effect on the fluid motion throughout the experiment by the imposed horizontal boundary conditions. The osmosis process may seem to be influenced by the fluid flow near the top and bottom boundaries. But, considering that the osmosis process is very fast in the salt–water solution by the very high osmotic pressure and that the flow is even slower at the boundaries, the effect does not seem to be appreciable.

4. CONCLUSIONS

Natural convection of a stratified salt–water solution due to lateral heating in a rectangular enclosure of aspect ratio 3.0, is experimentally investigated. Visual observation shows four distinct flow regimes depending on the buoyancy ratio, N, which represents the relative magnitude of solutal stratification to thermal buoyancy due to lateral heating. They are the unicell flow regime for $N < 10$, the simultaneously formed layer flow regime for $10 \leqslant N < 40$, the successively formed layer flow regime for $40 \leqslant N < 55$ and the stagnant flow regime for $N \geqslant 55$.

In the unicell flow regime, the temperature is stably stratified similar to that of pure thermal convection and the concentration distributions are uniform. In the stagnant flow regime, the temperature varies strictly in the horizontal direction while the initially given salt stratification remains almost unchanged. In the simultaneously formed layer flow regime the temperature in each layer varies much in the vertical direction while the concentration remains nearly equal. Both profiles are similar in the successively formed layer except the mid-region where the temperature profile is unstably stratified and the concentration is stably stratified. Due to the large difference of diffusivity, the temperature varies smoothly at the interface of each layer, but the concentration varies rapidly at the interface separating the fast moving layer while it varies smoothly at the interface of the slowly moving layer.

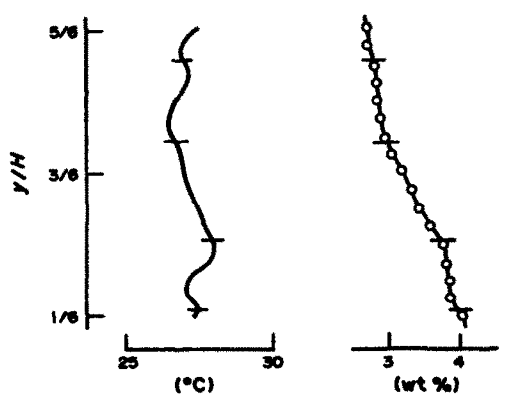


FIG. 4. Vertical temperature and concentration profiles at $x/L = 1/2$ for successively formed layer flow regime: $Ra_T = 9.20 \times 10^7$, $N = 50.1$.

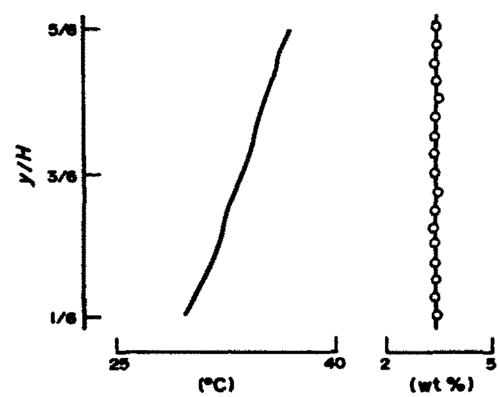
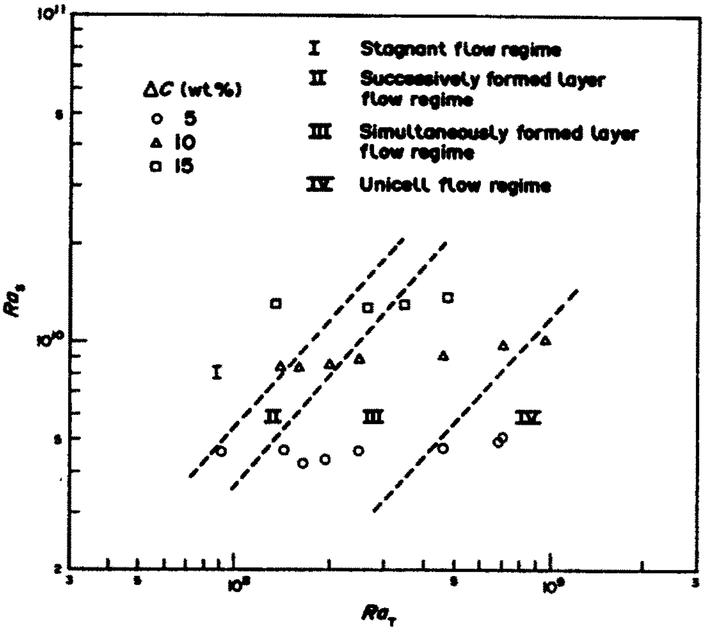


FIG. 6. Vertical temperature and concentration profiles at $x/L = 1/2$ for unicell flow regime: $Ra_T = 7.08 \times 10^8$. $N = 7.2$.



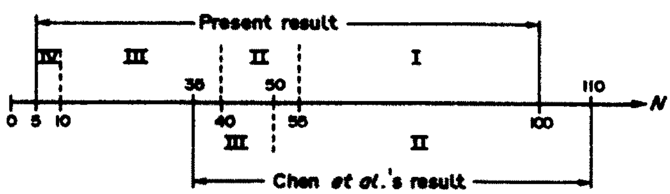


FIG. 7. Flow regimes depending on: (a) thermal and solutal Rayleigh numbers; (b) buoyancy ratio.

REFERENCES

1. S. Ostrach, Natural convection with combined driving forces, *PhysicoChem. Hydrodyn.* **1**, 233–247 (1980).
2. J. S. Turner and H. Stommel, A new case of convection in the presence of combined vertical salinity and temperature gradient, *Proc. Natn. Acad. Sci.* **52**, 49–53 (1964).
3. J. S. Turner, *Buoyancy Effects in Fluids*. Cambridge University Press, Cambridge (1979).
4. S. A. Thorpe, P. K. Hutt and R. Soulsby, The effect of horizontal gradients on thermohaline convection, *J. Fluid Mech.* **38**, 375–400 (1969).
5. C. F. Chen, D. G. Briggs and R. A. Wirtz, Stability of thermal convection in a salinity gradient due to lateral heating, *Int. J. Heat Mass Transfer* **14**, 57–65 (1971).
6. R. A. Wirtz, D. G. Briggs and C. F. Chen, Physical and numerical experiments on layered convection in a density stratified fluid, *Geophys. Fluid Dyn.* **3**, 265–288 (1972).
7. R. A. Wirtz, The effect of solute layering on lateral heat transfer in an enclosure, *Int. J. Heat Mass Transfer* **20**, 841–846 (1977).
8. F. P. Incropera and R. Viskanta, Optical studies of mixed layer development in a double-diffusive, thermohaline system, *Proc. 7th Int. Heat Transfer Conf.*, Vol. 2, pp. 419–424 (1982).
9. W. T. Lewis, F. P. Incropera and R. Viskanta, Interferomic study of stable salinity gradients heated from below or cooled from above, *J. Fluid Mech.* **116**, 411–430 (1982).
10. Y. Kamotani, L. W. Wang, S. Ostrach and H. D. Jiang, Experimental study of natural convection in shallow enclosures with horizontal temperature and concentration gradients, *Int. J. Heat Mass Transfer* **28**, 165–173 (1985).
11. J. Lee and M. T. Hyun, Experiments on thermosolutal convection in a shallow rectangular enclosure, *Expl Thermal Fluid Sci.* **1**, 259–265 (1988).
12. J. Lee, M. T. Hyun and K. W. Kim, Natural convection in confined fluids with combined horizontal temperature and concentration gradients, *Int. J. Heat Mass Transfer* **31**, 1969–1977 (1988).
13. M. E. Stern and J. S. Turner, Salt fingers and convection layers, *Deep-Sea Res.* **16**, 497–511 (1969).
14. M. E. Stern, Lateral mixing of water masses, *Deep-Sea Res.* **14**, 747–753 (1967).
15. G. Oster, Density gradients, *Scient. Am.* **213**, 70–76 (1965).

16. S. J. Khang and T. J. Fitzgerald, A new probe and circuit for measurement electrolyte conductivity, *Ind. Engng Chem. Fund.* **14**, 208–213 (1975).
17. W. H. Kaufmann, *Sodium Chloride*. Reinhold, New York (1960).
18. A. Bejan, A. A. Al-Homoud and J. Imberger, Experimental study of high Rayleigh number convection in a horizontal cavity with different end temperatures, *J. Fluid Mech.* **109**, 283–299 (1981).
19. S. H. Yin, T. Y. Wung and K. Chen, Natural convection in an air layer enclosure within rectangular cavities, *Int. J. Heat Mass Transfer* **21**, 307–315 (1978).

CONVECTION NATURELLE CONFINEE DUE AU CHAUFFAGE LATERAL DANS UNE SOLUTION STRATIFIEE STABLE

Résumé—On conduit des expériences pour étudier la convection naturelle, dans une solution stratifiée stable sel–eau, induite par un chauffage latéral dans une enceinte rectangulaire de rapport de forme 3,0. Selon le rapport de flottement N qui représente l'importance relative de la stratification solutale et du flottement thermique, quatre régimes distincts d'écoulement sont observés : régime d'écoulement unicellulaire pour $N < 10$, régime à couche limite simultanée pour $10 \leqslant N < 40$, régime d'écoulement à couche successivement formée pour $40 \leqslant N < 55$ et régime stagnant pour $N \geqslant 55$. La formation et la croissance de la structure de l'écoulement à couche limite sont observées et décrites, avec les distributions correspondantes de température et de concentration dans chaque couche.

BEGRENZTE NATÜRLICHE KONVEKTION AUFGRUND SEITLICHER ERWÄRMUNG IN EINER STABIL GESCHICHTETEN LÖSUNG

Zusammenfassung—In einem rechtwinkligen Behälter mit dem Seitenverhältnis 3,0 wurde die stationäre natürliche Konvektion aufgrund seitlicher Erwärmung einer stabil geschichteten Salzwasserlösung experimentell untersucht. Abhängig vom Auftriebsverhältnis N, das die Relation zwischen Schichtung und thermischem Auftrieb beschreibt, werden vier verschiedene Strömungsformen beobachtet: einzellige Strömung (für $N < 10$), gleichzeitiges Einsetzen der Schichtenströmung (für $10 \leqslant N < 40$), allmähliches Einsetzen der Schichtenströmung (für $40 \leqslant N < 55$) und Stillstand (für $N \geqslant 55$). Die Bildung und das Wachstum der geschichteten Strömung wird visuell beobachtet und durch die entsprechenden Temperatur- und Konzentrationsverteilungen in jeder Schicht beschrieben.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В ЗАМКНУТОМ ОБЬЕМЕ, ВЫЗВАННАЯ БОКОВЫМ НАГРЕВОМ УСТОЙЧИВО СТРАТИФИЦИРОВАННОГО РАСТВОРА

Аннотация—Экспериментально исследуется установившаяся естественная конвекция в устойчиво стратифицированном водосолевом растворе, вызванная боковым нагревом прямоугольной полости с отношением сторон 3,0. В зависимости от отношения подъемных сил N, представляющего величину отношения стратификации раствора к подъемной силе из-за разности температур, отчетливо различаются четыре режима течения—одноячеистого течения при $N < 10$, одновременно образующихся слоев течения при $10 \leqslant N < 40$, последовательно формирующихся слоев течения при $40 \leqslant N < 55$ и неизменяющегося течения при $N \geqslant 55$. Визуально наблюдалось и описано образование и рост ламинарной структуры течения с соответствующими распределениями температур и концентраций в каждом сдое.